1 a) $\psi(1,2) = \psi_1 \psi_2 = \sqrt{\frac{1}{4\pi^2}}\, e^{in_1\phi_1} e^{in_2\phi_2}$

(hiermee kunnen alle eigenfuncties geconstrueerd worden: bv. $\psi_1 + \psi_2$: algemene opl. $\sum_{nm} c_{nm} \psi_n \psi_m$. neem combinatie $n=0, m=m$ en $n=n, m=0 \Rightarrow \psi_m + \psi_n$)

$E = E_1 + E_2 = (n_1^2 + n_2^2) \frac{\hbar^2}{2ma^2}$

b) $\sqrt{\frac{2}{3\pi^2}} \cos^2\phi_1 e^{i\phi_2} = \sqrt{\frac{2}{3\pi^2}} \left(\frac{1}{2}\right)^2 (e^{i\phi_1} + e^{-i\phi_1})^2 e^{i\phi_2} =$

$\sqrt{\frac{2}{3\pi^2}} \frac{1}{4} (e^{2i\phi_1} + 2 + e^{-2i\phi_1}) e^{i\phi_2}$

$E = \frac{4\hbar^2}{2ma^2}$ $\quad E=0 \quad$ $E = \frac{4\hbar^2}{2ma^2}$ $\quad E = \frac{\hbar^2}{2ma^2}$

[$\psi(t) = e^{-iHt/\hbar} \psi(0) \overset{*}{=} e^{-iEt/\hbar} \psi(0)$
* als ψ is eigenfunctie van H]

$t = T$

$\psi(T) = \sqrt{\frac{2}{3\pi^2}} \frac{1}{4} \left(e^{2i\phi_1} e^{-i\frac{4\hbar^2}{2ma^2}\frac{T}{\hbar}} + 2 + e^{-2i\phi_1} e^{-i\frac{4\hbar^2}{2ma^2}\frac{T}{\hbar}} \right) e^{i\phi_2} e^{-i\frac{\hbar^2}{2ma^2}\frac{T}{\hbar}}$

normering : $\int_0^{2\pi} |\psi(T)|^2 dr = \frac{2}{3\pi^2} \frac{1}{4^2} \left((2\pi)^2 + 4(2\pi)^2 + (2\pi)^2 \right) = 1$

c) $\psi(T) = \sqrt{\frac{2}{3\pi^2}} \frac{1}{4} \left(e^{2i\phi_1} e^{-i\frac{5\hbar^2 T}{2ma^2 \hbar}} + 2e^{-i\frac{\hbar^2}{2ma^2}\frac{T}{\hbar}} + e^{-2i\phi_1} e^{-i\frac{5\hbar^2}{2ma^2}\frac{T}{\hbar}} \right) e^{i\phi_2}$

$E=5 \qquad E=1 \qquad E=5$

kans op $E=5$: $\frac{2}{3\pi^2} \frac{1}{4^2} \left((2\pi)^2 + (2\pi)^2 \right) = \frac{1}{3}$

kans op $E=1$: $\frac{2}{3\pi^2} \frac{1}{4^2} \left(4 (2\pi)^2 \right) = \frac{2}{3}$

2 a) $L^2 Y_{l,m} = l(l+1)\hbar^2 Y_{l,m}$ $\qquad$ Let op: $\frac{1}{\sqrt{8\pi}} \sim Y_{0,0}$

$\Rightarrow L^2 Y_{0,0} = 0$, $L^2 Y_{1,m} = 1(1+1)\hbar^2 Y_{1,m} = 2\hbar^2 Y_{1,m}$ $\qquad$ $Y_{l,m}$ zijn orthonormaal

b) $l=0$ deel $\int \left(\frac{1}{\sqrt{8\pi}}\right)^2 d\Omega = \frac{1}{8\pi} \cdot 4\pi = \frac{1}{2}$ $\qquad \langle L^2 \rangle = \frac{1}{2} \times 0 = 0$

$l=1$ deel $\int \left(\frac{1}{\sqrt{10}} [-i Y_{1,1} + \sqrt{7} Y_{1,0} + Y_{1,-1}] \right)^2 d\Omega$

$= \frac{1}{10}(1+7+1) = \frac{1}{2}$ $\qquad \langle L^2 \rangle = \frac{1}{2} \cdot l(l+1) = \frac{1}{2} \cdot 2 = 1$

$\Rightarrow$ TOTAAL $\langle L^2 \rangle = 1$

c) $L_z Y_{l,m} = m\hbar Y_{l,m} \Rightarrow L_z = -1, 0, 1$

d) $\langle L_z \rangle = \frac{1}{2} \times 0 + \frac{1}{10} \times 1 + \frac{7}{10} \times 0 + \frac{1}{10}(-1) = 0$

e) $L_z = 0$: $\psi = N\left(\frac{1}{\sqrt{8\pi}} + \sqrt{\frac{7}{10}} Y_{1,0} \right)$

integreren over de ruimte $\int |\psi|^2 d\Omega = N^2 \left(\frac{1}{2} + \frac{7}{10} \right) = N^2 \frac{16}{10} \equiv 1$

$N = \sqrt{\frac{10}{16}}$